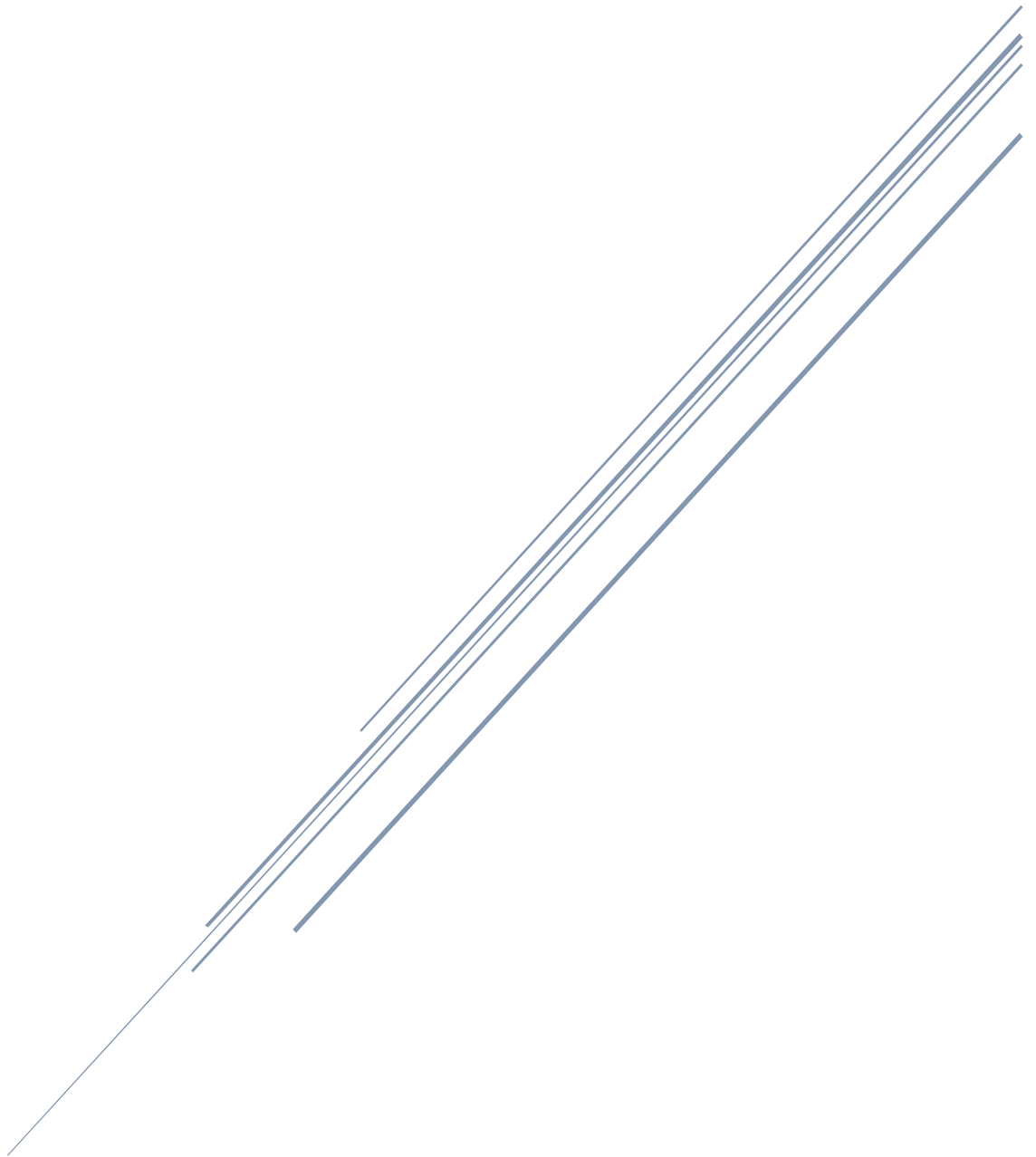


GEOMETRIC REASONING

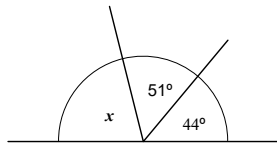
Using angle rules to determine unknown angles



Geometric Reasoning

Worked Example: Angles on a straight line total 180°

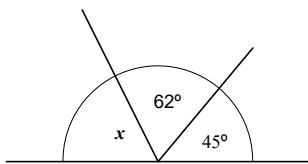
Find the value of x .



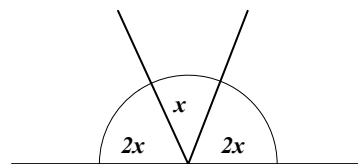
$$\begin{aligned} x &= 180 - (51 + 44) \text{ (Angles on a straight line total } 180^\circ\text{)} \\ &= 180 - 95 \\ &= 85^\circ \end{aligned}$$

Exercise: find the value of x on these straight lines.

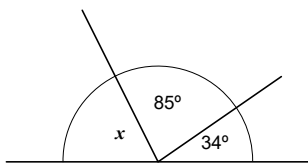
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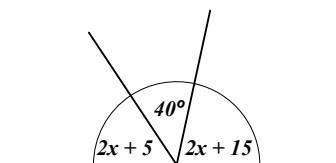
6



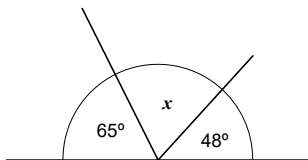
2



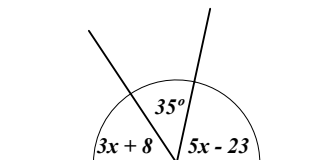
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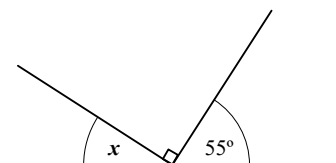
3



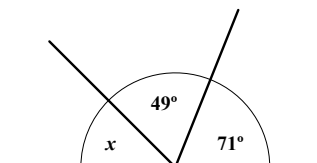
8



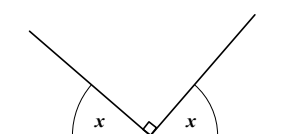
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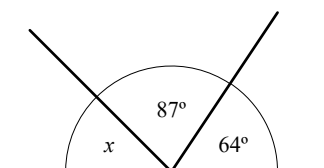
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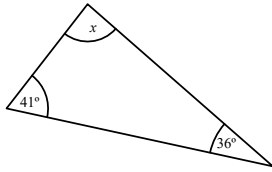
5



10



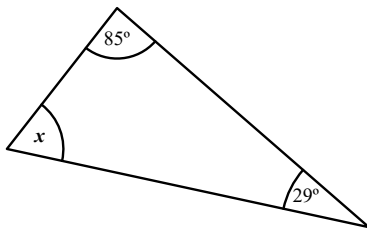
Worked Example: Angles in a triangle total 180°



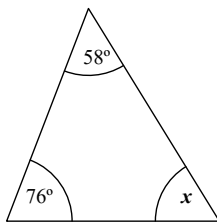
$$\begin{aligned}x &= 180 - (41 + 36) \text{ (Angles in a triangle total } 180^\circ\text{)} \\ &= 180 - 77 \\ &= 103^\circ\end{aligned}$$

Exercise: Find the value of x in these triangles

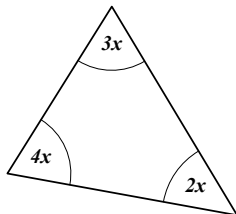
1



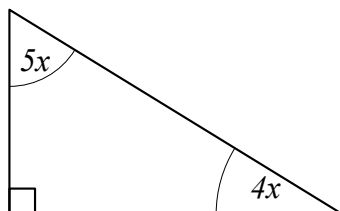
2



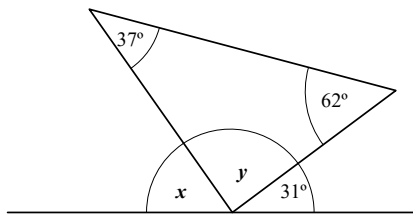
3



4



Worked Example: Combination of straight line and triangle

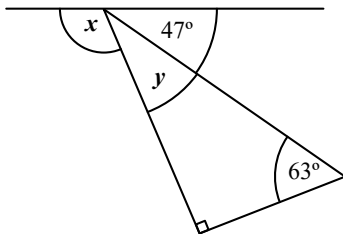


$$\begin{aligned} y &= 180 - (62 + 37) \text{ (Angles in a triangle total } 180^\circ\text{)} \\ &= 180 - 99 \\ &= 81^\circ \end{aligned}$$

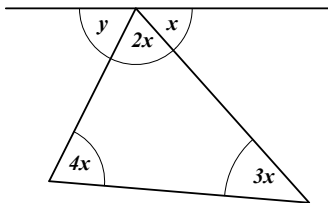
$$\begin{aligned} x &= 180 - (31 + 81) \text{ (Angles on a straight line total } 180^\circ\text{)} \\ &= 180 - 112 \\ &= 68^\circ \end{aligned}$$

Exercise on Combination of triangles and straight lines

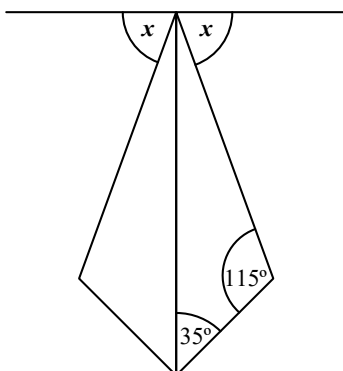
- 1 Find the size of angles x and y



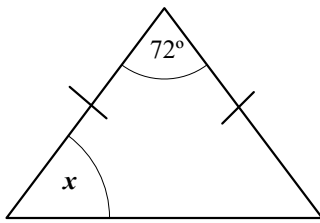
- 2 Find the size of angles x and y



- 3 The two triangles below are congruent. Find the size of angles x and y



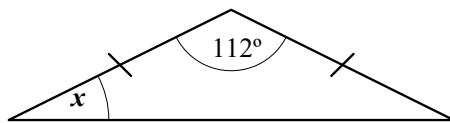
Worked Example: Base angles in an isosceles triangle are equal



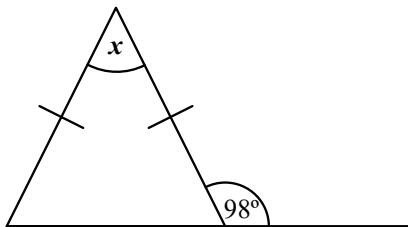
$$\begin{aligned}
 x &= \frac{180-72}{2} \quad (\text{Base angles in an isosceles triangle are equal}) \\
 &= \frac{108}{2} \\
 &= 54^\circ
 \end{aligned}$$

Exercise on triangles, straight lines, base angles in isosceles triangles

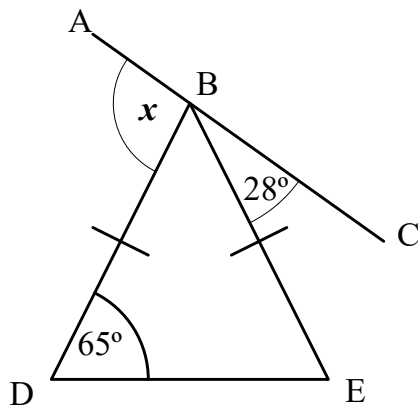
- 1 Find the size of angle x



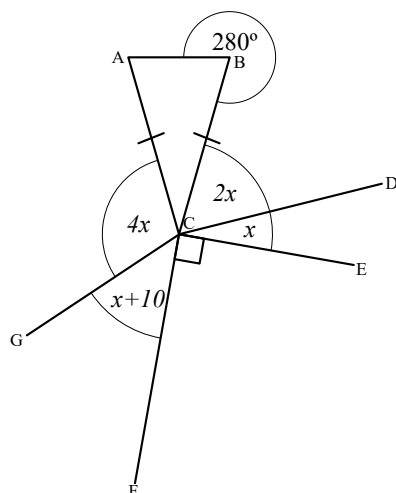
- 2 Find the size of angle x



- 3 Find the size of angle x



Worked Example: Angles about a point total 360°



$$\begin{aligned} ABC &= 360 - 280 \text{ (Angles about a point total } 360^\circ\text{)} \\ &= 80^\circ \end{aligned}$$

$$\begin{aligned} ACB &= 180 - (80 \times 2) \text{ (Base angles in an isosceles triangle are equal)} \\ &= 20^\circ \end{aligned}$$

$$ACB + BCD + ECD + FCE + GCF + ACG = 360^\circ \text{ (Angles about a point total } 360^\circ\text{)}$$

$$20 + 2x + x + 90 + x + 10 + 4x = 360$$

$$8x + 120 = 360$$

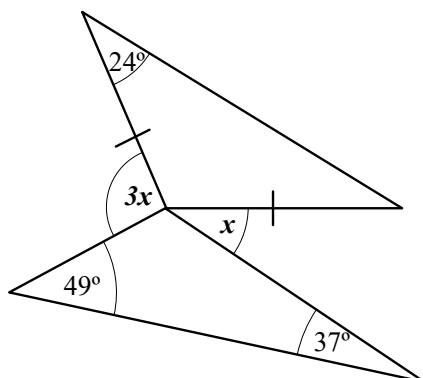
$$8x = 360 - 120$$

$$x = \frac{240}{8}$$

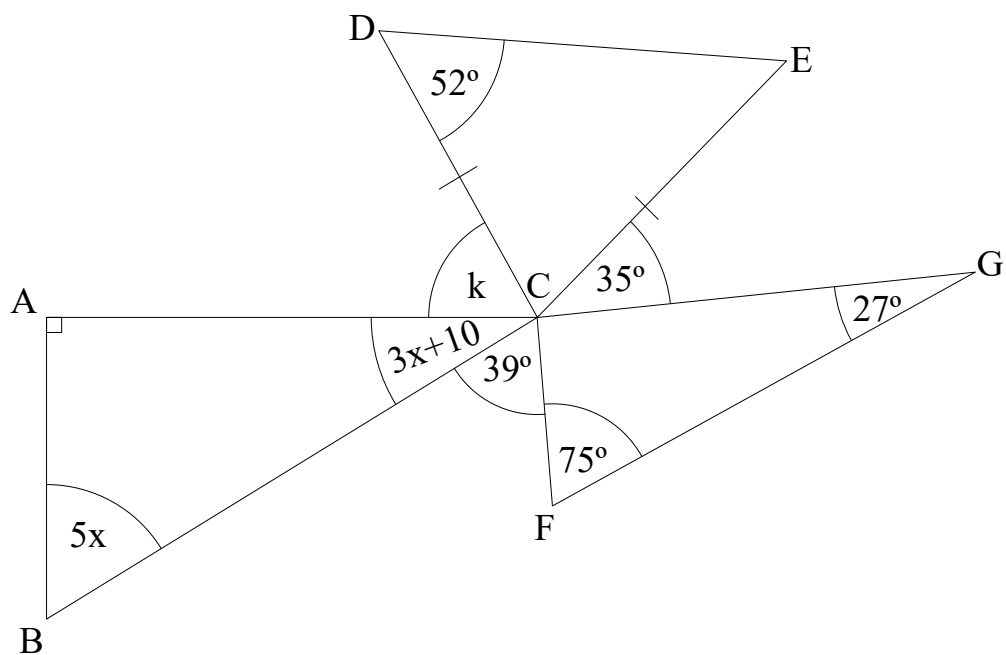
$$= 30$$

Exercise on angles about a point total 360°

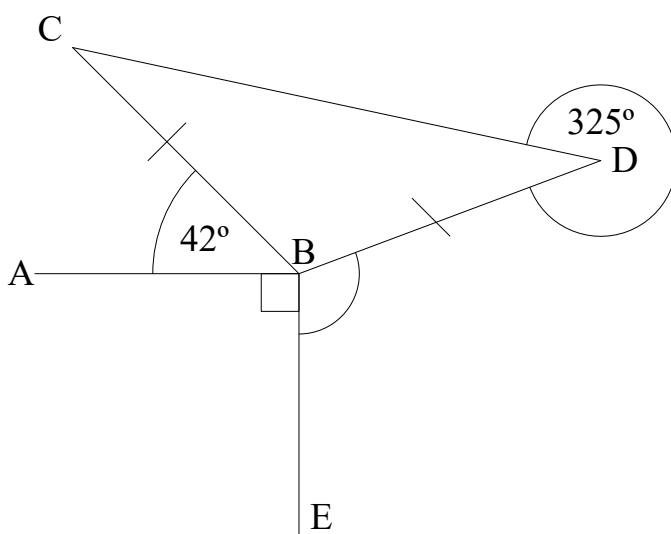
- 1 Find the size of angle x



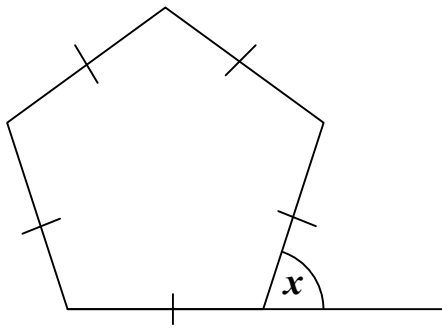
2 Find the size of angle ACD



3 Find the size of angle DBE



Worked Example on Exterior Angles

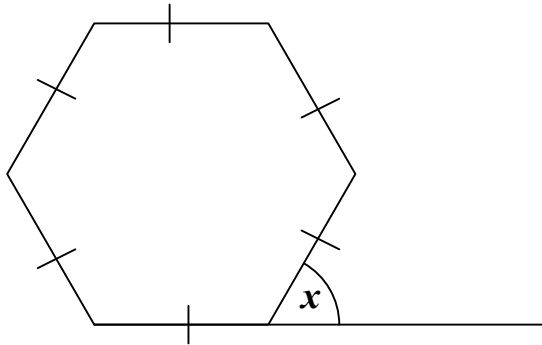


$$\begin{aligned} \text{Exterior angle} &= \frac{360}{\text{Number of sides}} \\ &= \frac{360}{5} \\ &= 72^\circ \end{aligned}$$

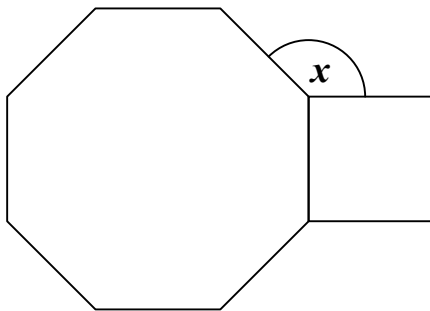
$$\text{So } x = 72^\circ$$

Exercise on exterior angles and rearranging the formula to find the number of sides

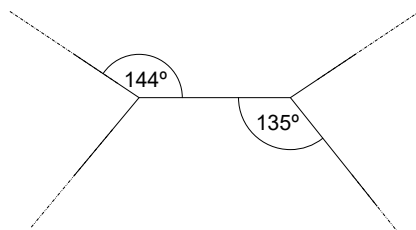
- 1 Find the value of x



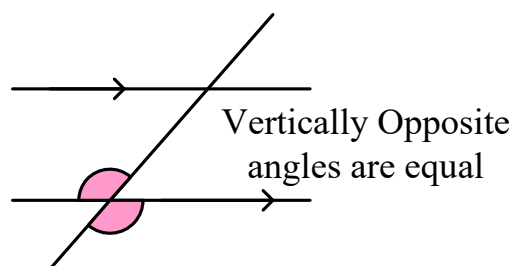
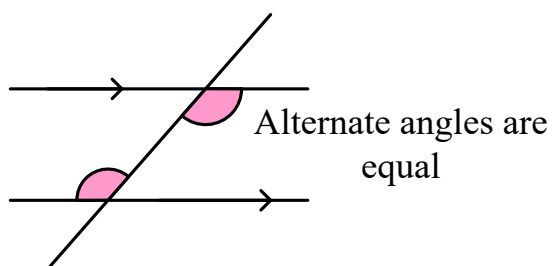
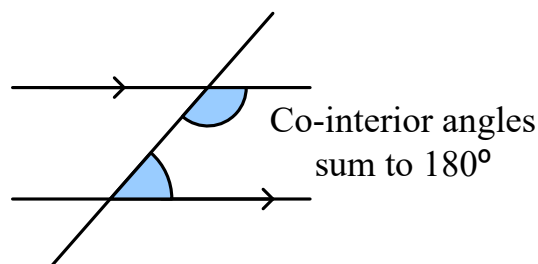
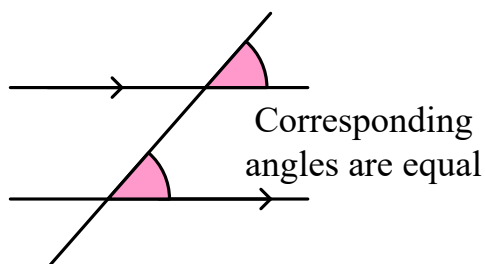
- 2 In the diagram below, all shapes are regular. Find the angle marked x .



- 3 Below is a partial view of two regular polygons. Name the polygons.

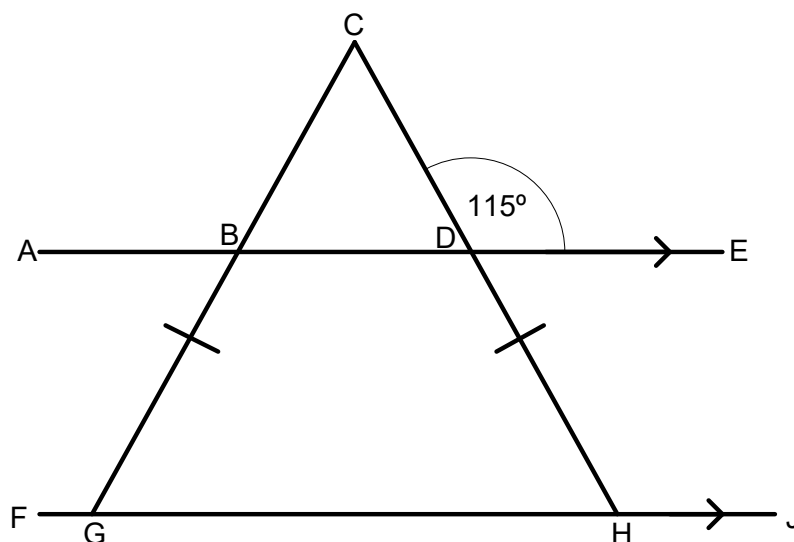


Parallel lines

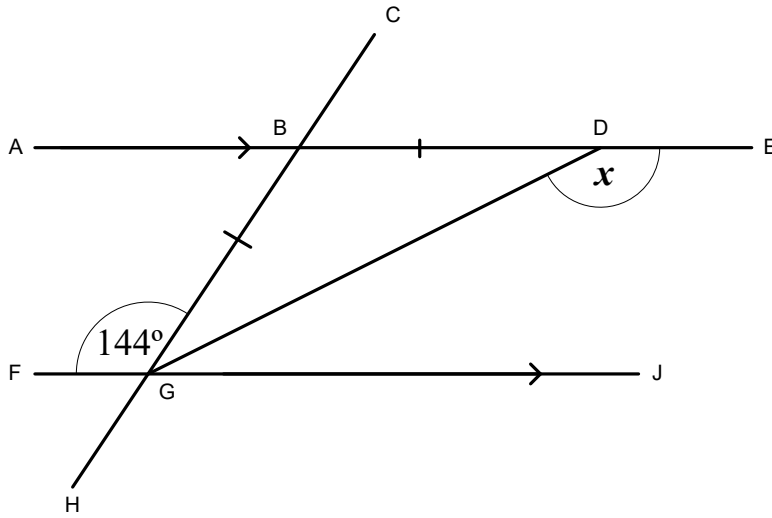


Exercises involving parallel lines

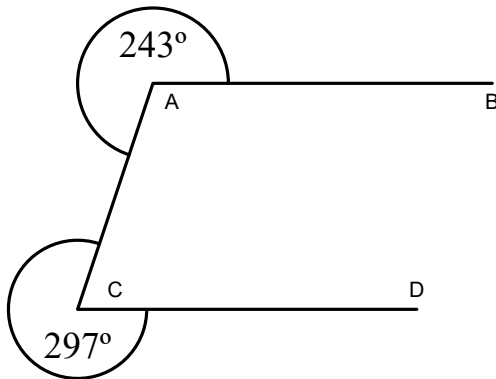
- 1 GCH is an isosceles triangle. For each angle in the diagram below:
 - a. Name it using 3 letter notation;
 - b. State the size of the angle;
 - c. State the reason you know.



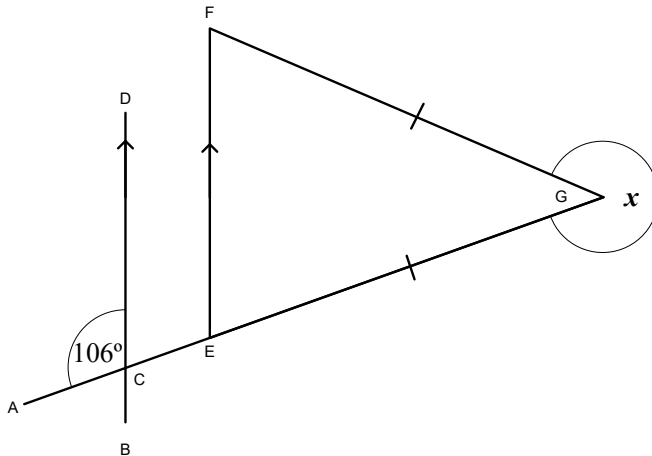
- 2 Find angle x . Give reasons for each stage.



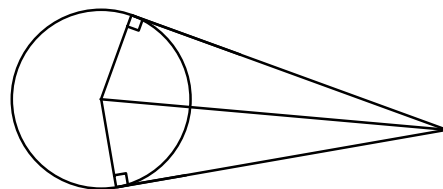
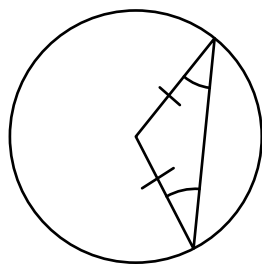
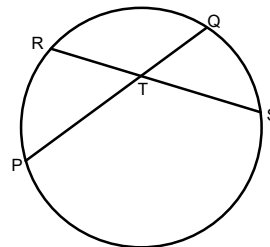
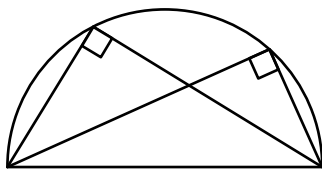
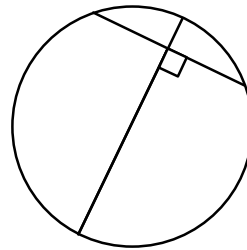
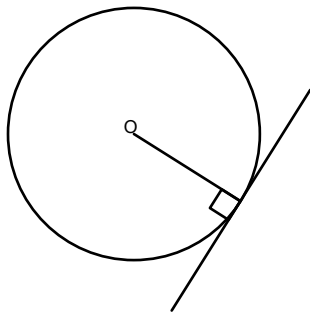
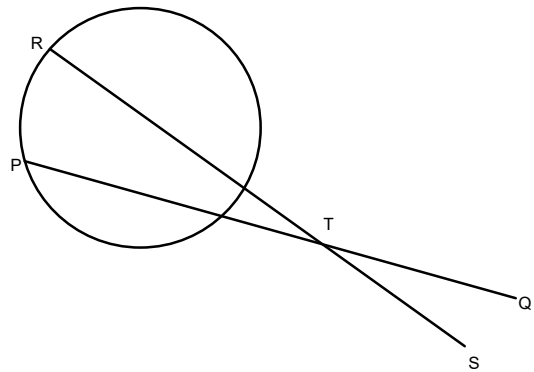
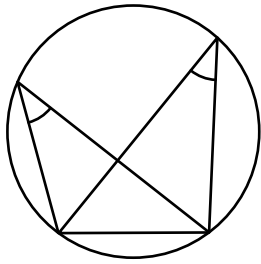
- 3 Bill says that AB is parallel to CD. Freda says that Bill is wrong. Who is correct? How do you know?

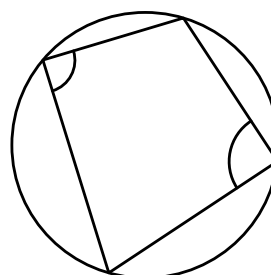
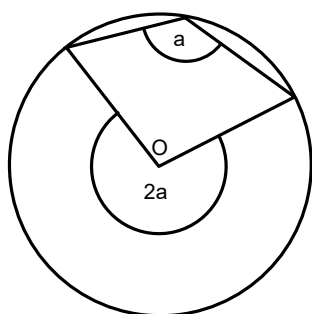
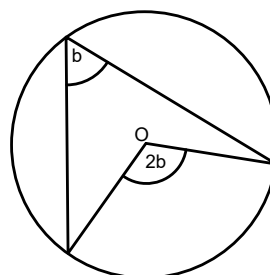
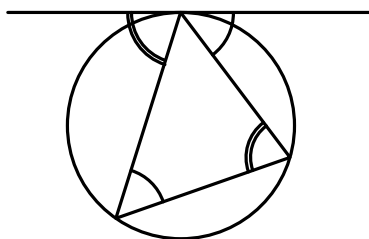


- 4 Find angle x .



Circle Theorems – match up the diagram with the text of each circle theorem





The angle in a semi-circle is a right angle.

A tangent to a circle makes a right angle with the radius at that point.

A triangle formed by two radii is isosceles except when the angle at the centre is 60° .

A diameter bisects a chord at right angles.

An angle subtended at the centre of a circle is double the angle subtended at the circumference by the same arc.

An angle subtended at the centre of a circle is double the angle subtended at the circumference by the same arc.

Angles subtended by an arc in the same segment are equal.

Opposite angles in a cyclic quadrilateral sum to 180° .

Two tangents drawn from a single point outside the circle are the same length and form two congruent triangles.

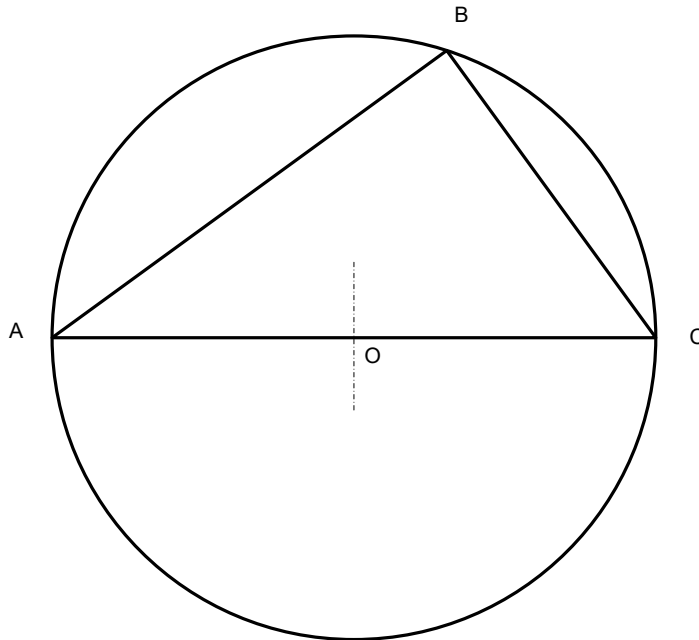
The angle between a tangent and a chord is equal to the angle subtended from the ends of the chord in the alternate segment.

If two chords, PQ and RS cross at point T then $PT \times QT = RT \times ST$

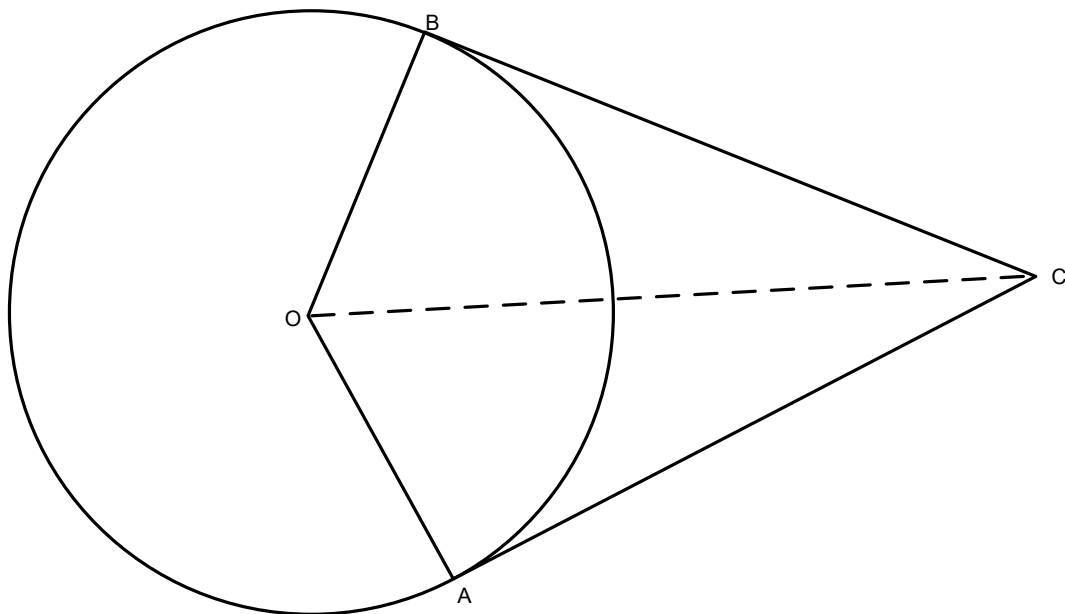
If two chords, PQ and RS cross at point T then $PT \times QT = RT \times ST$, even if T is outside the circle.

Proof of Circle Theorems

- 1 AOC is a diameter of a circle. Without resorting to circle theorems, prove that $\angle ABC$ is a right angle.

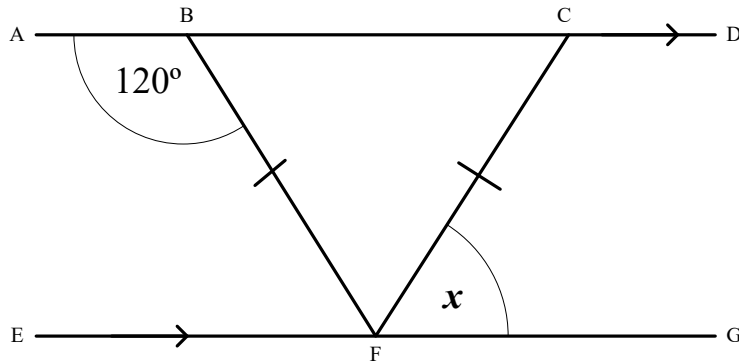


- 2 Prove that $AC=BC$

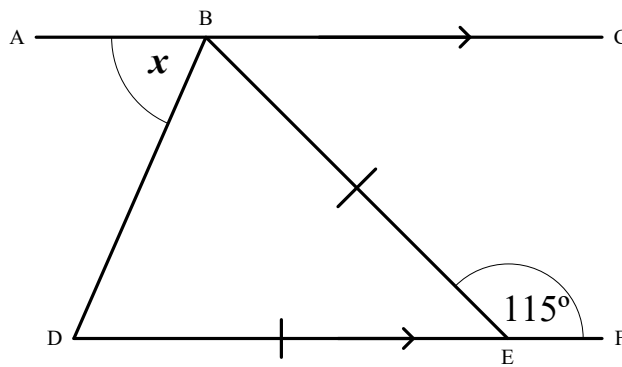


General Practice

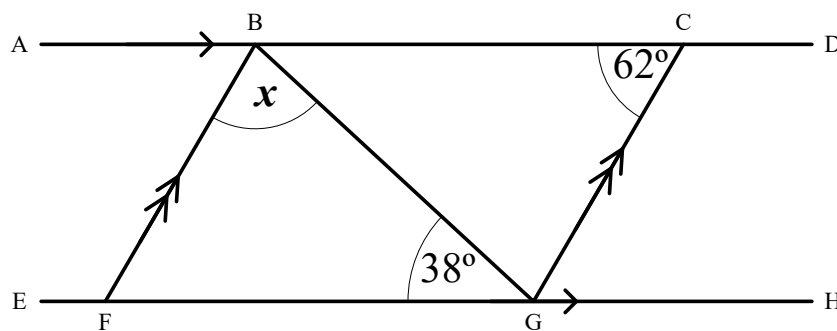
- 1 AD and EG are straight parallel lines.
BFC is an isosceles triangle.
Find $\angle CFG$. State your reasons at each stage of your working.



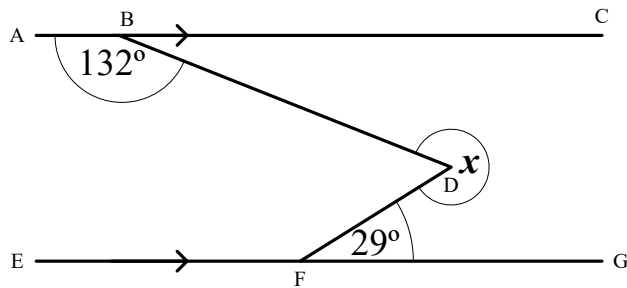
- 2 AC and DF are straight parallel lines.
DBE is an isosceles triangle.
Find $\angle ABD$. State your reasons at each stage of your working.



- 3 AD and EH are straight parallel lines.
FB and GC are straight parallel lines.
Angle BCG is 62° and angle BGF is 38° .
Find angle FBG. State your reasons at each stage of your working.



- 4 AC and EG are straight parallel lines.
 Angle ABD is 132° and angle DFG is 29° .
 Find the reflex angle FDB. State your reasons at each stage of your working.



- 5 AB is a line of symmetry.
 EFGH, CDHP and HJKL are squares.
 HLMNP is a regular pentagon.
 Find angle GHL. State your reasons at each stage of your working.

